MATHEMATICS METHODS

MAWA Semester 1 (Unit 3) Examination 2016

Calculator-free

Marking Key

Section One: Calculator-free (50 Marks)

Question 1(a)(i)

Solution	
$y = \left(\cos(x)\right)^{-\frac{1}{2}}$ $\frac{dy}{dx} = -\frac{1}{2}\left(-\sin(x)\right)\left(\cos(x)\right)^{-\frac{3}{2}}$	
Marking key/mathematical behaviours	Marks
• rewrites as a power • differentiates using chain rule	1 1

Question 1(a)(ii)

Solution	
$\frac{dy}{dx} = \frac{2e^{2x}(-\cos(1-x)) - 4e^{2x}\sin(1-x)}{\left(2e^{2x}\right)^2}$	
Marking key/mathematical behaviours	Marks
• correctly determines numerator of derivative • correctly determines denominator of derivative	1 1

Question 1(b)

Solution	
$\frac{dy}{dx} = 6x\left(2x+1\right)^5 + \left(-3x^2\right).5\left(2x+1\right)^4.2$ $= 6x\left(2x+1\right)^4\left[\left(2x+1\right)+5x\right]$ $= 6x\left(2x+1\right)^4\left(7x+1\right)$	
Marking key/mathematical behaviours	Marks
• correctly differentiates using product and chain rule • correctly factorises • correctly simplifies	1 1 1

Question 2(a)

Solution

$$\int \frac{x^3}{2} - x + 1 \, dx = \frac{x^4}{8} - \frac{x^2}{2} + x + c$$

Marking key/mathematical behaviours	Marks
• correctly integrates each term	1
• correctly adds constant of integration (1 mark penalty once only throughout the rest of question 2)	1

Question 2(b)

Solution

$$\int \frac{x^2 - 2}{\sqrt{x}} dx$$

$$= \int x^{\frac{3}{2}} - 2x^{-\frac{1}{2}} \, dx = \frac{2x^{\frac{5}{2}}}{5} - 4x^{\frac{1}{2}} + c$$

Marking key/mathematical behaviours	Marks
• correctly simplifies integral	1
• correctly integrates each term	1

Question 2(c)

Solution

$$\int 2x(x+1)^2 \, dx$$

$$= \int 2x^3 + 4x^2 + 2x \, dx = \frac{x^4}{2} + \frac{4x^3}{3} + x^2 + c$$

Marking key/mathematical behaviours	Marks
• correctly expands and simplifies integral	1
• correctly integrates each term	1

Question 2(d)

Solution

$$\int e^{\frac{x}{2}} - \cos\left(\frac{2x}{3}\right) dx = 2e^{\frac{x}{2}} - \frac{3}{2}\sin\left(\frac{2x}{3}\right) + c$$

Marking key/mathematical behaviours	Marks
• correctly integrates first term	1
• correctly integrates second term	1

Question 3(a)

Solution $d = 0.3$	
Marking key/mathematical behaviours	Marks
• determines correct value	1

Question 3(b)

Solution 4(0.1) + 5(0.3) + 6(0.3) + 7(0.2) + 8(0.1) = 5.9	
Marking key/mathematical behaviours	Marks
• adds the products of y with the probability that it will occur • determines the correct value for the expected value	1 1

Question 3(c)

Solution (i) 0.6 (ii) 0.5 (iii) $\frac{6}{9}$	
Marking key/mathematical behaviours	Marks
• obtains correct value • obtains correct value • obtains correct value	1 1 1

Question 4(a)

Solution $f''(x) = 3(2x)(2x+6)(x^2+1)^2 + 2(x^2+1)^3$	
Marking key/mathematical behaviours	Marks
• determines the first part of the derivative using the product rule • determines the second part of the derivative using the product rule	1 1

Question 4(b)

Solution $f''(-3) = 2000$	
Marking key/mathematical behaviours	Marks
• determines the value of the second derivative at $x = -3$	1

Question 4(c)

Solution Since $f'(-3) = 0$ and $f''(-3) = 2000 > 0$ the point is a local minimum.	
Marking key/mathematical behaviours	Marks
• determines $f'(-3)$ • states the point is a local minimum.	1 1

Question 5(a)

Solution

(i) E(H) = E(X) + 3 = 30

(ii) Var (H) = Var (X) = 25

Marking key/mathematical behaviours	Marks
• calculates correct value of E(H)	1
• calculates correct value of Var(H)	1

Question 5(b)

Solution

(i) E(G) =2 E(H) = 2(30) = 60

(ii) standard deviation of G = 2 × standard deviation of H = 10

Marking key/mathematical behaviours	Marks
• calculates correct value of E(G)	1
• calculates correct value of the standard deviation of H	1

Question 6

Solution

$$\frac{d^2y}{dx^2} = 3\sqrt{2x-3} - 2$$

$$\frac{dy}{dx} = (2x-3)^{\frac{3}{2}} - 2x + c_1 \Rightarrow 4 = (4)^{\frac{3}{2}} - 7 + c_1 \Rightarrow c_1 = 3$$

$$y = \frac{1}{5}(2x-3)^{\frac{5}{2}} - x^2 + 3x + c_2 \Rightarrow -\frac{4}{5} = \frac{1}{5} - 4 + 6 + c_2 \Rightarrow c_2 = -3$$

$$\therefore \quad y = \frac{1}{5}(2x-3)^{\frac{5}{2}} - x^2 + 3x - 3$$

Marking key/mathematical behaviours	Marks
• correctly determines first derivative	1
• correctly determines the value of c_1	1
• correctly determines y	1
• correctly determines the value of c_2 and writes y in terms of x	1

Question 7

Solution

$$\frac{dy}{dx} = \frac{(2x-1)^2(1) - 4(2x-1)(x+1)}{(2x-1)^4}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{1(1) - 2(4)}{1}$$

$$= -7$$

$$y = -7x + c$$

$$8 = -7(1) + c$$

$$c = 15$$

$$y = -7x + 15$$

Marking key/mathematical behaviours	Marks
• correctly determines the numerator of the derivative using the quotient rule	1
• correctly determines the denominator of the derivative using the quotient rule	1
• correctly determines the gradient of the curve at $(1,8)$	1
• correctly substitutes the point $(1,8)$ into the equation to evaluate c	1
• correctly determines the equation of the tangent	1

Question 8 (a)

Solution

$$\left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

Marking key/mathematical behaviours	Marks
• determines correct probability	1

Question 8(b)

Solution

$$\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right) = \frac{2}{27}$$

Marking key/mathematical behaviours	Marks
• determines correct probability	1

Question 8(c)

Solution

$$\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 \times 3 = \frac{4}{9}$$

Marking key/mathematical behaviours	Marks
• correctly multiplies by three	1
• determines correct probability	1

Question 8(d)

Solution

$$1 - \left(\frac{2}{3}\right)^3 = \frac{19}{27}$$

Marking key/mathematical behaviours	Marks
• recognises complementary events	1
• determines correct probability	1

Question 9(a)

Solution

$$\int_{\frac{\pi}{6}}^{\pi} \cos(3x)\ dx = \left[\frac{\sin 3x}{3}\right]_{\frac{\pi}{6}}^{\pi}$$

$$= -\frac{1}{3}$$

Marking key/mathematical behaviours	Marks
• correctly integrates	1
• correctly evaluates	1

Question 9(b)

Solution

$$\frac{d}{dx}\left(\int_2^x \sqrt{3 - 2t^2}\, dt \right) = \sqrt{3 - 2x^2}$$

Marking key/mathematical behaviours	Marks
• correctly applies fundamental theorem	1

Question 9(c)

Solution

$$\int_0^2 \frac{d}{dx}\left(\frac{1-x^2}{1+x}\right)\,dx = \left[\frac{1-x^2}{1+x}\right]_0^2$$

$$= \qquad -1-1=-2$$

Marking key/mathematical behaviours	Marks
• correctly integrates	1
• correctly evaluates	1